Commission scolaire English-Montréal
English Montreal School Board

Mathematics — 565-426

Secondary IV

May 2015

May Practice Exam

Competency Two
Uses Mathematical Reasoning

Science Option

ANSWER KEY

ANSWER KEY FOR THE EXAMINATION

PARTS A AND B

Part A	**Questions 1 to 6**	
	4 marks or 0 marks	

1. B | 4 | 0 |

2. D | 4 | 0 |

3. D | 4 | 0 |

4. A | 4 | 0 |

5. D | 4 | 0 |

6. C | 4 | 0 |

Part B	**Questions 7 to 10**	
	4 marks or 0 marks	

7. The distance between the Frisbee and the ground decreased for **17** seconds. | 4 | 0 |

8. The length of line segment EF is **312** cm.. | 4 | 0 |

9. The diameter of the cylinder is **8** cm. | 4 | 0 |

10. The solutions of the inequality are **the values belonging to the interval]0, 9[**. | 4 | 2 | 0 |

Note: Accept equivalent answers such as "**the values in between 0 and 9**".

Give 2 marks if the student wrote the two critical values (i.e. 0 and 9).

PART C

11. ANGLE EFD

EXAMPLE OF AN APPROPRIATE SOLUTION

➢ LENGTH OF LINE SEGMENTS CE AND DE AND BD

$\cos(m\angle EFB) = \dfrac{m\overline{BF}}{m\overline{BE}}$ { Cosine ratio in right triangle BFE

$\cos 40^o = \dfrac{18cm}{m\overline{BE}}$ → $m\ \overline{BE} = \dfrac{18cm}{\cos 40^o}$ = 23.4973......m ≈ 23.5 cm

$m\ \overline{CE} = m\ \overline{BE} - m\ \overline{BC}$ = 23.5 cm – 14 cm = 9.5 cm

$m\ \overline{DE} = m\ \overline{CE}$ = 9.5 cm

$m\ \overline{BD} = m\ \overline{BE} + m\overline{DE}$ = 23.5 cm + 9.5 cm = 33 cm

➢ LENGTH OF LINE SEGMENT DF

$(m\ \overline{DF})^2 = (m\ \overline{BD})^2 + (m\ \overline{BF})^2 - 2(m\ \overline{BD})(m\ \overline{BF})\cos(m\angle FBD)$ { Cosine law applied in triangle BFD

$(m\ \overline{DF})^2 = (33\text{ cm})^2 + (18\text{ cm})^2 - 2(33\text{ cm})(18\text{ cm})\cos 40^o$
$(m\ \overline{DF})^2 = 1089\text{ cm}^2 + 324\text{ cm}^2 - 910.0608\text{ cm}^2$
$(m\ \overline{DF})^2 = 502.9392\text{ m}^2$
$m\ \overline{DF} = 22.4263...\text{m}$

➢ MEASURE OF ANGLE DFE

$m\angle FBE + m\angle BFE + m\angle BEF = 180^o$ {sum of angles in a triangle
$40^o + 90^o + m\angle BEF = 180^o$ → $m\angle BEF = 50^o$

$m\angle FED + m\angle BEF = 180^o$ {the angles are supplementary
$m\angle FED + 50^o = 180^o$ → $m\angle FED = 130^o$

$\dfrac{m\overline{ED}}{\sin(m\angle DFE)} = \dfrac{m\overline{DF}}{\sin(m\angle DEF)}$ { Sine law applied in triangle EFD

$\dfrac{9.5cm}{\sin(m\angle DFE)} = \dfrac{22.4cm}{\sin 130^o}$ → $\sin(m\angle DFE) = 0.3248..$ → $m\angle DFE = 18.9586^o$

➢ CONCLUSION

To the nearest degree, the measure of angle DFE is 19 ^{o}.

12. TWO PARABOLAS

EXAMPLE OF AN APPROPRIATE SOLUTION

- **RULE OF FUNCTION *f***

$f(x) = a(x-h)^2 + k$

$f(x) = a(x-30)^2 + 12.5$ Using the vertex (30, 12.5)

$0 = a(40-30)^2 + 12.5$ Using $f(40) = 0$

$-12.5 = a\ (100)$

$-0.125 = a$

Rule of function f : $f(x) = -0.125(x-30)^2 + 12.5$

- **Y-INTERCEPT OF FUNCTIONS *f* AND *g***

$f(0) = -0.125(0-30)^2 + 12.5$

$f\ (0) = -0.125(900) + 12.5$

$f\ (0) = -100$

$g\ (0) = f\ (0) = -100$

- **RULE OF FUNCTION *g***

Function g passes through (0, –100), (10, 0) and (40, 0).

$g(x) = a\ (x-10)\ (x-40)$ Using the two zeros 10 and 40

$-100 = a\ (0-10)\ (0-40)$ Using $g\ (0) = -100$

$-100 = a\ (400)$

$-0.25 = a$

Rule of function g : $g(x) = -0.25\ (x-10)\ (x-40)$

- **VERTEX OF FUNCTION *g***

The x-coordinate the vertex of function g: $\frac{10+40}{2} = 25$

$g\ (25) = -0.25\ (25-10)\ (25-40)$

$g\ (25) = -0.25\ (15)\ (-15) = 56.25$

The coordinates of the vertex of function g are (25, 56.25).

- **DISTANCE BETWEEN THE TWO VERTICES**

$d\ (V_f, V_g) = \sqrt{(30-15)^2 + (12.5-56.25)^2} = \sqrt{225 + 1914.0625} = 46.25 \approx 46.3$ units

- **CONCLUSION**

To the nearest tenth of a unit, the distance between the vertices of the parabolas is 46.3 units.

13. A PENTAGON

EXAMPLE OF AN APPROPRIATE SOLUTION

- **VALUE OF X**

Area of triangle CDG = $\dfrac{m\overline{GC} \times m\overline{GD}}{2}$

$= \dfrac{(5x+3)(8x-1)}{2} = \dfrac{40x^2+19x-3}{2}$

Area of rectangle AFDE = Area of triangle GCF {Since equivalent plane figures are equal in area.

$6x^2+41x+30 = \dfrac{40x^2+19x-3}{2}$

$12x^2 + 82x + 60 = 40x^2 + 19x - 3$

$0 = 28x^2 - 63x - 63$

$7(4x^2 - 9x - 9) = 0$

$7(x - 3)(4x - 3) = 0$

$x = 3$ or $x = -\dfrac{3}{4}$ {this value is impossible because the length of line segment GC would be less than zero.

Thus, x is equal to 3.

- **NUMERICAL VALUES OF THE LENGTH OF LINE SEGMENTS GC AND GF**

Length of line segment GC = 8(3) – 1 = 23 units

Area of rectangle AFDE = $m\overline{AF} \times m\overline{FD}$

$6x^2 + 41x + 30 = (x + 6)(6x + 5)$

$m\overline{FD} = 6x + 5$

Length of line segment GF = $m\overline{FD} - m\overline{GD}$

Length of line segment GF = $(6x + 5) - (5x + 3) = x + 2$

Length of line segment GF = 3 + 2 = 5 units

- **AREA OF RECTANGLE FBCG**

Area of rectangle FBCG = $m\overline{GC} \times m\overline{GF}$

Area of rectangle FBCG = 23 u × 5 u

Area of rectangle = 115 u^2

- **CONCLUSION**

The numerical value of the area of rectangle FBCG is115 u^2.

14. THE JEWELLERY PACKAGES

EXAMPLE OF AN APPROPRIATE SOLUTION

- **SYSTEM OF EQUATIONS REPRESENTING THE INFORMATION ON THE MASS OF THE JEWELLERY PLACED IN PACKAGES A AND B**

x: mass of a bracelet, in grams
y: mass of a necklace, in grams
$20x + 25y = 4500$
$64x + 48y = 9600$

- **MASS OF A BRACELET AND MASS OF A NECKLACE**

$20x + 25y = 4500 \xrightarrow{\times 48} 960x + 1200y = 216\,000$
$64x + 48y = 9600 \xrightarrow{\times 25} -\underline{(1600x + 1200y = 240\,000)}$
$-640x = -24000$
$x = 37.5$

$20(37.5) + 25y = 4500$
$750 + 25y = 4500$
$25y = 3750$
$y = 150$ Mass of a bracelet : 37.5 g Mass of a necklace : 150 g

- **MASS OF PACKAGE C**

Total mass of package C: 32 × 37.5 g + 96 × 150 g = 15 600 g
Mass of all three packages together : 4500 g + 9600 g + 15 600 g = 29 700 g

- **COST OF SHIPPING**

Cost of shipping package A:

$f(4500) = -\frac{3}{2}\left[-\frac{4500}{500}\right] + 20$

$f(4500) = -\frac{3}{2}[-9] + 20$

$f(4500) = 13.50 + 20$

$f(4500) = \$33.50$

Cost of shipping package B:

$f(9600) = -\frac{3}{2}\left[-\frac{9600}{500}\right] + 20$

$f(9600) = -\frac{3}{2}[-19.2] + 20$

$f(9600) = -\frac{3}{2}(-20) + 20$

$f(9600) = \$50$

Cost of shipping package C:

$f(15\,600) = -\frac{3}{2}\left[-\frac{15600}{500}\right] + 20$

$f(15\,600) = -\frac{3}{2}[-31.2] + 20$

$f(15\,600) = -\frac{3}{2}(-32) + 20$

$f(15\,600) = \$68$

Cost of shipping 3 packages together:

$f(29\,700) = -\frac{3}{2}\left[-\frac{29700}{500}\right] + 20$

$f(29\,700) = -\frac{3}{2}[-59.4] + 20$

$f(29\,700) = -\frac{3}{2}(-60) + 20$

$f(29\,700) = \$110$

Cost of A + B + C separately : \$33.50 + \$50 + \$68 = \$151.50 > \$110

- **CONCLUSION**

If the jewellery maker combined all three packages into one package, the shipping cost would be less than the sum of the shipping cost of the three packages separately.

15. THE GARDEN

EXAMPLE OF AN APPROPRIATE SOLUTION

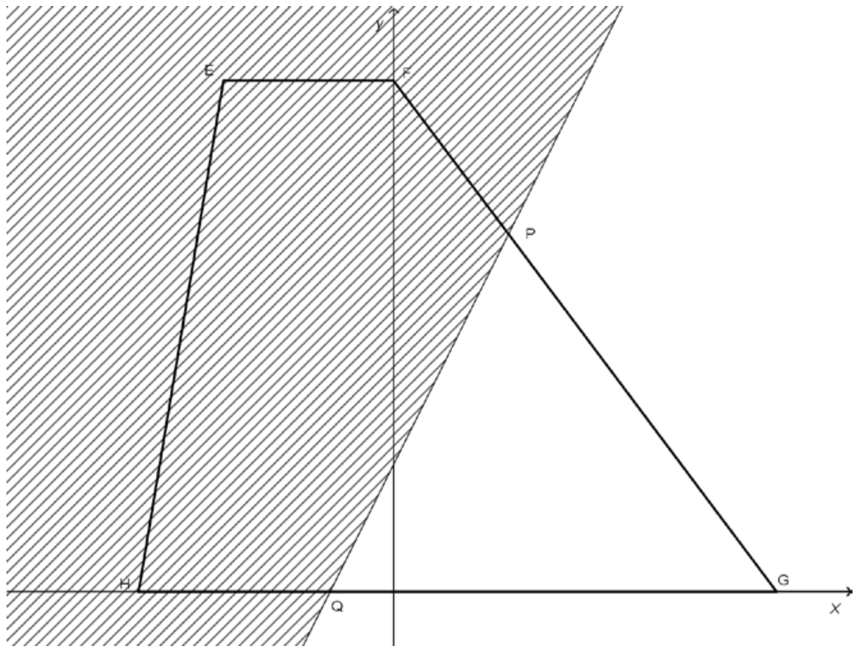
- **REGION WHERE VEGETABLES WILL BE PLANTED**

Equation of the line defining the half-plane:

$$\frac{x}{-15}+\frac{y}{30}\geq 1$$

Or $y \geq 2x + 30$
Test point (0, 0)
So, $0 \geq 2(0) + 30$.
$0 \geq 30$? False.

Let P be the point of intersection of the line defining the half-plane and line segment FG.
Let Q be the point of intersection of the line defining the half-plane and line segment GH.



Pentagon EFPQH represents the region where the vegetables will be planted.

- **COORDINATES OF POINTS P AND Q**

Equation of line FG:

$$a=\frac{0-120}{90-0}=-\frac{4}{3} \quad \rightarrow \quad y=-\frac{4}{3}x+120 \text{ or } 4x+3y=360$$

Coordinates of point P by solving the system of equations:
$4x + 3(2x + 30) = 360$
$10x + 90 = 360 \rightarrow 10x = 270 \rightarrow x = 27$
$y = 2(27) + 30 = 84$

Coordinates of point P: (27, 84)
Coordinates of point Q: (–15, 0)

- **AREA OF THE PORTION OF THE GARDEN WHERE TINA WILL PLANT VEGETABLES**

Area of pentagon EFPQH = Area of trapezoid – Area of triangle PGQ
Area = area of trapezoid – area of triangle PGQ

$$\text{Area} = \frac{(150+40)\times 120}{2}-\frac{m\overline{GQ}\times(\text{distance\_between\_P\_and\_}\overline{GQ})}{2}$$

$$\text{Area} = \frac{(150+40)\times 120}{2}-\frac{105\text{dm}\times 84\text{dm}}{2}$$

Area = 11400 dm^2 – 4410 dm^2 = 6990 dm^2

- **CONCLUSION**

The area of the portion of the garden where Tina will plant vegetables this year is 6990 dm^2.

16. LINE SEGMENT DF

EXAMPLE OF AN APPROPRIATE SOLUTION

➢ EQUATION OF LINE SEGMENT BE

Slope of $\overline{BE}$ × slope of $\overline{AB} = -1$ — Since line segments BE and AB are perpendicular

Slope of $\overline{BE} \times -\frac{3}{4} = -1$

Slope of $\overline{BE} = \frac{4}{3}$ — So, $y = \frac{4}{3}x + b$

Using B (360, –384), $-384 = \frac{4}{3}(360) + b$

$-864 = b$ — The equation of line segment BE is $y = \frac{4}{3}x - 864$

➢ LENGTHS OF LINE SEGMENTS BE AND AB

Coordinates of point E : $0 = \frac{4}{3}x - 864$ $\quad 864 = \frac{4}{3}x \quad \rightarrow \quad x = 648$

Coordinates of point E : (648, 0)

$m\,\overline{BE} = \sqrt{(648-360)^2 + (0-(-384))^2}$ = 480 units

$\tan\,(m\angle AEB) = \frac{m\overline{AB}}{m\overline{BE}}$ — {Tangent ratio applied in right triangle ABE

$\tan 22.62^\circ = \frac{m\overline{AB}}{480} \quad \rightarrow \quad m\,\overline{AB}$ = 200 units

➢ SIMILARITY OF TRIANGLES ABD AND CDF

AB // DF	because lines with equal slopes are parallel.
$\angle ABC \cong \angle CDF$	because two alternate interior angles formed by two parallel lines (AB and DF) cut by a transversal ($\overline{BD}$) are congruent.
$\angle ACB \cong \angle DCF$	because vertically opposite angles are congruent

If two angles of one triangle and the two corresponding angles of another triangle are congruent, then the triangles are similar. Therefore, Δ FGH ~ Δ JKG.

➢ LENGTH OF LINE SEGMENT DF

In similar figures, the lengths of corresponding segments are proportional.

$\frac{m\overline{DF}}{m\overline{AB}} = \frac{m\overline{CD}}{m\overline{BC}} \quad \rightarrow \quad \frac{m\overline{DF}}{200} = \frac{275}{125} \quad \rightarrow \quad m\overline{DF}$ = 440 units

➢ CONCLUSION

$m\overline{DF}$ = 440 units

Appendix C