Commission scolaire English-Montréal
English Montreal School Board

Mathematics — 565-426

Secondary IV

May 2014

May Practice Exam

Competency Two
Uses Mathematical Reasoning

Science Option

Administration and Marking Guide

ANSWER KEY FOR THE EXAMINATION
PARTS A AND B

Part A	Questions 1 to 6
	4 marks or 0 marks

1. A — 4 | 0

2. B — 4 | 0

3. C — 4 | 0

4. D — 4 | 0

5. C — 4 | 0

6. C — 4 | 0

Part B	Questions 7 to 10
	4 marks or 0 marks

7. The height of the pyramid is **80** cm. — 4 | 0

8. The binomial **8*x*** – **6** represents the result of the division. — 4 | 0

9. The cost of mailing a package weighing 4.7 kilograms is **$12**. — 4 | 0

10. The solutions of the inequality are **the values belonging to the interval**
] –∞, –15] U [–5, + ∞[. — 4 | 2 | 0

Note: Accept equivalent answers such as "**the values less than or equal to –15 as well as the values greater than or equal to –5**".

Give 2 marks if the student wrote the two critical values (i.e. –15 and –5).

PART C

11. HIGH SCHOOL PLAYGROUND

EXAMPLE OF AN APPROPRIATE SOLUTION

➢ LENGTH OF LINE SEGMENT AC

$\cos(m\angle BAC) = \dfrac{m\overline{AB}}{m\overline{AC}}$ { Cosine ratio in right triangle ABC

$\cos 28^o = \dfrac{36\text{m}}{m\overline{AC}}$

$m\ \overline{AC} = \dfrac{36\text{m}}{\cos 28^o} = 40.7725.....\text{m}$

➢ LENGTH OF LINE SEGMENT CD

$\dfrac{m\overline{CD}}{\sin(m\angle CED)} = \dfrac{m\overline{DE}}{\sin(m\angle ECD)}$ { Sine law applied in triangle CDE

$m\ \angle CED + m\ \angle CDE + m\ \angle ECD = 180^o$

$m\ \angle CED + 30^o + 100^o = 180^o$ → $m\ \angle CED = 50^o$

$\dfrac{m\overline{CD}}{\sin 50^o} = \dfrac{68.4\text{m}}{\sin 100^o}$ → $m\ \overline{CD} = 53.2057...\text{m}$

➢ LENGTH OF LINE SEGMENT AD

$(m\ \overline{AD})^2 = (m\ \overline{AC})^2 + (m\ \overline{CD})^2 - 2\ (m\ \overline{AC})(m\ \overline{CD})\cos(m\ \angle ACD)$ { Cosine law applied in triangle ACD

$(m\ \overline{AD})^2 = (40.77\text{ m})^2 + (53.21\text{ m})^2 - 2\ (40.77\text{ m})(53.21\text{ m})\cos 42^o$

$(m\ \overline{AD})^2 = 1662.1929\text{ m}^2 + 2831.3041\text{ m}^2 - 3224.3147\text{ m}^2$

$(m\ \overline{AD})^2 = 1269.1823\text{ m}^2$

$m\ \overline{AD} = 35.6255...\text{m}$

➢ CONCLUSION

The length of the fence to be replaced is 35.6 m.

12. THE WATERCRAFT

EXAMPLE OF AN APPROPRIATE SOLUTION

- **TIME WHEN WATERCRAFT ENTERS WATER (ZEROS OF FUNCTION *f*)**

$f(x) = -1.6x^2 + 3.2x$
$0 = -1.6x^2 + 3.2x$
$0 = -1.6x\ (x - 2)$
$x = 0$ and $x = 2$ — The watercraft enters the water after 2 seconds.

Zeros of function *f*

- **RULEOF FUNCTION *g***

Since the watercraft stays underwater for 6 seconds, it emerges to the surface after 8 seconds (2 + 6 = 8). Zeros of function *g* : 2 and 8

$g(x) = a\ (x - 2)\ (x - 8)$
Using $g\ (3) = 1.0$
$1.0 = a\ (3 - 2)\ (3 - 8)$
$1 = a\ (1)\ (-5)$
$-0.2 = a$

Rule of function *g* : $g(x) = -0.2\ (x - 2)\ (x - 8)$

- **MAXIMUM DISTANCE UNDERWATER OF WATERCRAFT (VERTEX OF FUNCTION *g*)**

The *x*-coordinate the vertex of function *g*: $\frac{2+8}{2} = 5$
$g\ (5) = -0.2\ (5 - 2)\ (5 - 8)$
$g\ (5) = -0.2\ (3)\ (-3) = -1.8$

Maximum distance underwater of watercraft: -1.8 m

- **CONCLUSION**

The maximum distance underwater of the watercraft is -1.8 m on this test run.

13. DANCE HALL

EXAMPLE OF AN APPROPRIATE SOLUTION

➢ **LENGTH OF LINE SEGMENT JK**

$m\overline{JK} = \sqrt{(52-13)^2 + (13-65)^2} = \sqrt{4225} = 65$ units

➢ **CONGRUENCE OF ANGLES FGH AND JKG**

FG // JK	because lines with equal slopes are parallel.
$\angle FGH \cong \angle JKG$	because two corresponding angles formed by two parallel lines (FG and JK) cut by a transversal ($\overline{KH}$) are congruent.

➢ **CONGRUENCE OF TRIANGLES FGH AND JKG**

$\angle FGH \cong \angle JKG$	(Proven in previous step.)
$m\overline{JK} = m\overline{HG} = 65$ units	(According to the first step and the given information.)
$\angle FHG \cong \angle GJK$	because they are right angles according to the given information.

If two angles andf the contained side of one triangle are congruent to the corresponding two angles and contained side of another triangle, then the triangles are congruent.
Therefore, Δ FGH ≅ Δ JKG.

➢ **CONGRUENCE OF LINE SEGMENTS FG AND GK**

Since the corresponding segements in congruent figures are congruent, $\overline{FG} \cong \overline{GK}$.

➢ **CONCLUSION**

$m\overline{FG} = m\overline{GK}$

14. THE FARM FIELD

EXAMPLE OF AN APPROPRIATE SOLUTION

- **EQUATION OF LINE MN**

$a = \frac{0-70}{168-0} = -\frac{5}{12}$ → $y = -\frac{5}{12}x + 70$

- **REGION WHERE CARROTS WILL BE GROWN**

$35x - 12y - 5040 \geq 0$
x-intercept: (144, 0);
y-intercept: (-420, 0)
Let test point be (0, 0).
So, 35(0) – 12 (0) – 5040 ≥ 0.
- 5040 ≥0 ? False.

Inequality intersects line MN at P
$35x - 12\left(-\frac{5}{12}x + 70\right) - 5040 = 0$
$40x - 5880 = 0 \quad x = 147$
$35(147) - 12y - 5040 = 0$
$5145 - 12y - 5040 = 0$
$105 = 12y \quad y = 8.75$
Point P: (147, 8.75)

Inequality intersects line LM at Q
$35(168) - 12y - 5040 = 0$
$5880 - 12y - 5040 = 0 \quad 840 = 12y$ → $y = 70$ Point Q: (168, 70)

- **AREA OF THE PORTION OF THE FIELD WHERE NICHOLAS WILL GROW CARROTS**

Area = area of triangle PQM
Area = $\frac{m\overline{QM} \times (\text{distance between P and } \overline{QM})}{2}$
m $\overline{QM}$ = 70 – 0 = 70 m
Distance between P and $\overline{QM}$ = 168 – 147 = 21 m
Area = $\frac{70\text{m} \times 21\text{m}}{2}$
Area = 735 m^2

- **CONCLUSION**

The area of the portion of the field where Nicholas will grow carrots this year is 735 m^2.

15. REWARD POINTS

EXAMPLE OF AN APPROPRIATE SOLUTION

- **COMPARISON OF THE NUMBER OF POINTS JESSICA AND MAGGIE EARNED ACCORDING TO THE TOTAL COST ON GROCERIES**

First possibility

Maggie purchased a total of \$50 of groceries; therefore, Jessica purchased a total of \$150 of groceries.

Number of points Maggie earned:

$f(50) = -100\left[-\frac{1}{10}(50-2)\right]$

$f(50) = -100\left[-4.8\right]$

$f(50) = -100(-5)$

$f(50) = 500$

Number of points Jessica earned:

$f(150) = -100\left[-\frac{1}{10}(150-2)\right]$

$f(150) = -100\left[-14.8\right]$

$f(150) = -100(-15)$

$f(150) = 1500$

Observation: $500 \times 3 = 1500$
Jessica earned three times as many points as Maggie.

Second possibility

Maggie purchased a total of \$75 of groceries; therefore, Jessica purchased a total of \$225 of groceries.

Number of points Maggie earned:

$f(75) = -100\left[-\frac{1}{10}(75-2)\right]$

$f(75) = -100\left[-7.3\right]$

$f(75) = -100(-8)$

$f(75) = 500$

Number of points Jessica earned:

$f(225) = -100\left[-\frac{1}{10}(225-2)\right]$

$f(225) = -100\left[-22.3\right]$

$f(225) = -100(-23)$

$f(225) = 2300$

Observation: $800 \times 3 \neq 2300$
Jessica did not earn three times as many points as Maggie.

- **CONCLUSION**

☐ Jessica's statement is true.
☑ Jessica's statement is false.

Explanation

There is at least one example that contradicts Jessica's statement.

Note: Students can conclude that Jessica's statement is false after giving only one counter-example.

16. THE AREA RUG

EXAMPLE OF AN APPROPRIATE SOLUTION

➢ SYSTEM OF EQUATION REPRESENTING THE PERIMETER AND AREA OF THE RUG

x : side length of the shapes, in metres.
y : width of the grey rectangle, in metres.

Perimeter of rug : $8x + 4y + 4 = 42$
Area of rug : $3x^2 + 2xy + 2x = 68$ or $x(3x + 2y + 2) = 68$

➢ SIDE LENGTH OF SHAPES AND WIDTH OF GREY RECTANGLE

$8x + 4y + 4 = 42 \rightarrow y = -2x + 9.5$

By substitution, we obtain:
$3x^2 + 2x(-2x + 9.5) + 2x = 68$
$3x^2 - 4x^2 + 19x + 2x = 68$
$0 = x^2 - 21x + 68$
$0 = (x - 4)(x - 17)$

$x = 4$ or $x = 17$

If $x = 4$, $y = -2(4) + 9.5 = 1.5$
If $x = 17$, $y = -2(17) + 9.5 = -24.5$ (The width of the grey rectangle cannont be less than 0)

Side length of the shapes : 4 m
Width of the grey rectangle : 1.5 m

➢ AREA OF ONE GREY RECTANGLE

Area = 4 m × 1.5 m
Area = 6 m^2

➢ CONCLUSION

The area of one of the grey rectangle is 6 m^2.

Appendix C